Question 3) Cumulative and Conditional Probability

Homework Problems

Using the example problem as reference, complete the following five problems. Turn in the work and results for each of the five problems. (Note: 4 includes a,b,c and 5 includes a,b,c,d). Note that each problem provides a hint which should guide you through the problem along with the Example Problem as a reference. Post any questions you have to the General Discussion area.

**Problem 1:** You are taking a 10 question multiple choice test. If each question has four choices and you guess on each question, what is the probability of getting one question correct? *[Hint: This is a binomial in the form of 10 choose 1 with p=.25.]*

Solution: Getting one question correct out of 10 questions when guessing is “*10 choose 1*” *with p=.25.*

* Using the binomial equation – (10!/(10-1)!\*1!)\*.25^1\*(1-.25)^10-1

=10\*.25\*0.07508468627=0.1877

We calculate this as **0.1877**chance of getting one question correct out of 10 questions when guessing.

**Problem 2:** What is the probability of getting seven questions correct?

Solution 2: Getting 7 questions correct out of 10 questions when guessing is “*10 choose 7*” *with p=.25.*

Using the binomial equation – (10!/(10-7)!\*7!)\*.25^7\*(1-.25)^10-7

=120\*.25^7\*0.421875=0.0030

We calculate this as **0.0030** chance of getting 7 questions correct out of 10 questions when guessing.

**Problem 3:** What are your chances of answering seven questions correctly if you can reliably eliminate one possible answer from each question?

*Solution 3: Chances of answering seven questions correctly by* eliminating one possible answer from each question is is “*10 choose 7*” *with p=.33.*

Using the binomial equation – (10!/(10-7)!\*7!)\*.33^7\*(1-.33)^10-7

=120\*.33^7\*0.300763=0.0162

We calculate this as 0.0162chance of getting 7 questions correct out of 10 questions *by* eliminating one possible answer from each question.

**Problem 4:** Let's say, instead, that the test is an adaptive test; you get to answer more questions based on your previous success.

This test is structured like this:

* First you have to answer three questions and if you are correct on two of them, you get to answer three more questions.
* If two of **those** are correct, then you get three final questions, of which you need to get at least two correct to pass the whole test.

The test details are:

* The first test, T1, has three multiple choice questions with four possible answers each (*p*=0.25 per question).
* The second test, T2, has three multiple choice questions with three possible answers each (*p*=0.33 per question).
* The final test, T3, has three questions that are true/false (*p*=0.50 each question).

The test questions are formed as follows:

* The questions are in a language you have never seen: a mixture of Navaho, Swahili, Klingon, and Esperanto. So you have to guess on all of the questions and there are no contextual clues to eliminate any answers. This is the first one:

*'Arlogh Qoylu'pu'?* Moja: Yel kholgo eeah. Mbili: Floroj kreskas ĉirkaŭ mia domo. Pe'el! Tatu: La sandviĉo estos manĝota'mo'tlhIngan maH! Nne: 'Adeez'æ`q eeah. (The professor sits at the front of class with a giant, sadistic grin while the students throw wads of paper at his head.)

Using the binomial probability rule, the law of total probability and Bayes' theorem:

a) What is the probability of getting two right on each sub-exam? (T1, T2, and T3, separately.)

Solution 4 (a): The probability of getting two right on T1 - Test 1 sub-exam is “3 *choose 2*” *with p=.25.*

Using the binomial equation – (3!/(3-2)!\*2!)\*.25^2\*(1-.25)^3-2

=3\*.25^2\*.75=0.1406

We calculate this as 0.1406chance of getting two right on T1 sub-exam.

* The probability of getting two right on T2 - Test 2 sub-exam is “3 *choose 2*” *with p=.33.*

Using the binomial equation – (3!/(3-2)!\*2!)\**.33*^2\*(1-*.33*)^3-2

=3\*.*33*^2\*.67=0.218

We calculate this as **0.218** chance of getting two right on T2 sub-exam.

The probability of getting two right on T3 - Test 3 sub-exam is “3 *choose 2*” *with p=0.50.*

Using the binomial equation – (3!/(3-2)!\*2!)\**.50*^2\*(1-*.50*)^3-2

=3\*.*50*^2\*.50=0.375

We calculate this as 0.375chance of getting two right on T3 sub-exam.

b) What are your overall chances of passing the entire exam?

Solution b: Overall probability of passing the entire exam:

Table of contributions to total probability:

|  | **P** | **% contribution** |
| --- | --- | --- |
| p(T1) | 0.1406 | .333 |
| p(T2) | 0.218 | .333 |
| P(T3) | .375 | .333 |

Using the **law of total probability**:

Total probability of passing the entire exam(0.1406\*.33)+(0.218\*.33)+(.375\*.33)=0.242088

c) What are your chances of passing T3 if you first pass T1 and T2?

**Using Bayes' theorem :**

P(Pass T3|T1 and T2)= P(Pass T3)X P(Pass T|T2)x P(Pass T2)/((P(Pass T1)xP(Pass T2))

P(T1|T2)=P(Pass T1 and T2)/P(Pass T2)

=(0.1406\*0.218)/0.218=0.1406

P(Pass T3|T1 and T2)=(.375\*0.1406\*0.218)/(0.1406\*0.218)=0.375.

**Problem 5**: Now, let's say that you know just enough of these obscure languages to translate the first question in T1:

What time is it? (Klingon)

1. (Swahili): From dawn to setting sun. (Navajo)
2. (Swahili): Flowers grow around my house (Esperanto) so all of you may come in. (Klingon)
3. (Swahili): The sandwich will be eaten (Esperanto) because we are Klingons! (Klingon)
4. (Swahili): It's mid-afternoon. (Navajo) **[correct answer]**

Now the probability of passing T1 has changed because you only have to guess correctly on one of the two remaining questions in the first section, a one-in-two chance.

a) What is the new probability for T1?

The new probability for T1 is 0.5 (50%) because you only need to guess correctly on one of the two remaining questions in the first section. Which is a one-in-two chance.

b) Now what is the overall probability of passing the entire test?

Table of contributions to total probability:

|  | **P** | **% contribution** |
| --- | --- | --- |
| p(T1) | .50 | .333 |
| p(T2) | 0.218 | .333 |
| P(T3) | .375 | .333 |

Using the **law of total probability**:

The total probability of passing the entire test is (0.5\*.33)+(0.218\*.33)+(.375\*.33)=0.36069

c) And what is the probability of passing section T3, given that you have already passed sections T1 and T2?

Using Bayes' theorem:   
P(Pass T3|T1 and T2)= P(Pass T3)X P(Pass T|T2)x P(Pass T2)/((P(Pass T1)xP(Pass T2))

P(T1|T2)=P(Pass T1 and T2)/P(Pass T2)

=(0.5\*0.218)/0.218=0.5

The probability of passing section T3, given that you have already passed sections T1 and T2 is P(Pass T3|T1 and T2)=(.375\*0.5\*0.218)/(0.5\*0.218)=0.375

d) The kicker: How do you explain the difference between 4c and 5c? Can you relate this to a larger context about conditional probability and making decisions?

Both calculations yield the same result. This is because the requirement for T1 has changed to 0.5, but the relationship between T1 and T2 has not.